Find the logarithmic formula for $\tanh^{-1} x$ by solving $x = \tanh y$ for y **SCORE: ____ / 25 PTS**

using the exponential definition and an algebraic substitution $z = e^y$ (or a similar substitution).

$$x = \frac{e^y - e^{-y}}{e^y + e^{-y}} \quad 3$$

$$x = \frac{z - \frac{1}{z}}{z + \frac{1}{z}} \cdot \frac{z}{z} \quad 3$$

$$x = \frac{z^2 - 1}{z^2 + 1} \quad 3$$

$$xz^2 + x = z^2 - 1 \quad 3$$

$$x + 1 = z^2 - xz^2$$

$$x + 1 = z^2(1 - x) \quad 3$$

$$z^2 = \frac{1+x}{1-x} \quad 2$$

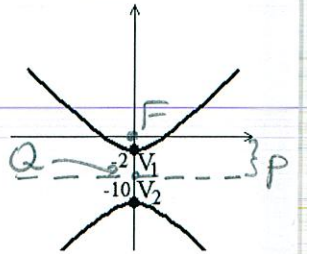
$$e^{2y} = \frac{1+x}{1-x} \quad 3$$

$$2y = \ln\frac{1+x}{1-x} \quad 3$$

$$y = \frac{1}{2}\ln\frac{1+x}{1-x} \quad 2$$

$$\tanh^{-1} x = \frac{1}{2}\ln\frac{1+x}{1-x}$$

The hyperbola on the right has a focus at the pole. The $y-$ coordinates of the vertices V_1 and V_2 are labeled. **SCORE: ____ / 25 PTS**



[a] Find polar co-ordinates for V_1, using **negative** values of r and θ. You do NOT need to show work.

$(2, \frac{3\pi}{2}) \longrightarrow (-2, \frac{3\pi}{2} - \pi - 2\pi) = (-2, -\frac{3\pi}{2})$ 3

[b] Find polar co-ordinates for V_2, using **positive** values of r and θ. You do NOT need to show work.

$(10, \frac{3\pi}{2})$ 2

[c] Find the **polar** equation of the hyperbola.

$e = \frac{V_1F}{V_1Q} = \frac{2}{p-2}$ $\quad e = \frac{V_2F}{V_2Q} = \frac{10}{10-p}$

3 $\frac{2}{p-2} = \frac{10}{10-p}$ 3

$20 - 2p = 10p - 20$

$40 = 12p$

$p = \frac{10}{3}$ 2

$e = \frac{2}{\frac{10}{3} - 2} \cdot \frac{3}{3} = \frac{6}{10-6} = \frac{6}{4} = \frac{3}{2}$ 2

$r = \left| \frac{ep}{1 - e\sin\theta} \right|$ 3

$r = \left| \frac{\frac{3}{2} \cdot \frac{10}{3}}{1 - \frac{3}{2}\sin\theta} \right|$ 3 $\cdot \frac{2}{2}$ 2

$r = \left| \frac{10}{2 - 3\sin\theta} \right|$ 2

MJ and NJ were working on their polar graphing partner quiz. **SCORE: _____ / 40 PTS**

On the question about the polar equation $r = 4\sqrt{3} + 8\sin 4\theta$, they determined correctly that the symmetry tests $(r, \pi - \theta)$, $(-r, \pi - \theta)$ and $(-r, -\theta)$ do **NOT** indicate that the graph is symmetric.

$\theta = \frac{\pi}{2}$ POLAR AXIS $\theta = \frac{\pi}{2}$

[a] **Using their results, along with the tests and shortcuts shown in lecture**, test if the graph is symmetric over the pole, the polar axis and/or $\theta = \frac{\pi}{2}$. State your conclusions in the table. **NOTE: Run as FEW tests as needed to prove your answers are correct.**

POLAR AXIS: $(r, -\theta)$

$r = 4\sqrt{3} + 8\sin 4(-\theta)$ 3

$r = 4\sqrt{3} - 8\sin 4\theta$ 2

POLE: $(r, \pi+\theta)$

$r = 4\sqrt{3} + 8\sin 4(\pi+\theta)$ 3

$r = 4\sqrt{3} + 8\sin(4\pi + 4\theta)$

$r = 4\sqrt{3} + 8[\sin 4\pi \cos 4\theta + \cos 4\pi \sin 4\theta]$ 2

$\sin 4\pi = 0$, $\cos 4\pi = 1$

$r = 4\sqrt{3} + 8\sin 4\theta$ 2

Type of symmetry	Conclusion
Over the pole	SYMMETRIC
Over the polar axis	NO CONCLUSION
Over $\theta = \frac{\pi}{2}$	NO CONCLUSION

$4 = 2+1+1$

[b] What is the minimum interval of the graph you need to plot first, before using reflections to draw the rest of the graph ?

$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ OR $[0, \pi]$ 4

[c] **Algebraically** find the angles in the minimum interval in part [b] at which the graph goes through the pole.
NOTE: You will NOT receive credit for just plugging in numbers to guess the answers.

$4\sqrt{3} + 8\sin 4\theta = 0$ 4

$\sin 4\theta = -\frac{\sqrt{3}}{2}$ 4

USING $[0, \pi]$: $0 \le \theta \le \pi$

$0 \le 4\theta \le 4\pi$

$4\theta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$ 8

$\theta = \frac{7\pi}{24}, \frac{11\pi}{24}, \frac{19\pi}{24}, \frac{23\pi}{24}$ 4

Find parametric equations for the line through $(-3, -6)$ and $(7, -2)$ using templates from your homework. **SCORE: ____ / 10 PTS**

$x = -3 + (7-(-3))t \longrightarrow x = -3+10t$ 5

$y = -6 + (-2-(-6))t \longrightarrow y = -6+4t$ 5

OR

$x = 7 + (-3-7)t \longrightarrow x = 7-10t$

$y = -2 + (-6-(-2))t \longrightarrow y = -2-4t$

Eliminate the parameter for the parametric equations $\begin{aligned} x &= 2\ln 3t \\ y &= 45t^4 \end{aligned}$.

SCORE: ____ / 15 PTS

For your final answer, solve for y in terms of x.

2 $\frac{x}{2} = \ln 3t$

4 $e^{\frac{x}{2}} = 3t$

2 $t = \frac{1}{3}e^{\frac{x}{2}}$

$y = 45\left(\frac{1}{3}e^{\frac{x}{2}}\right)^4$ 3

$y = 45 \cdot \frac{1}{81}e^{2x}$ 2

$y = \frac{5}{9}e^{2x}$ 2

Name the shape of the graphs of the following polar equations. Be as specific as possible. **SCORE: ____ / 20 PTS**

If the graph is a rose curve, state the number of petals.

[a] $r = \pi$ CIRCLE 2

[b] $r = \dfrac{7}{4 - 9\cos\theta}$ HYPERBOLA 3

[c] $r = \dfrac{9}{7 + 4\sin\theta}$ ELLIPSE 3

[d] $r = 9 + 4\cos\theta$ 2 CONVEX LIMACON 2

[e] $r = 7 - 9\sin\theta$ 2 LIMACON W/ LOOP 2

[f] $r = 7\sin 4\theta$ 2 ROSE CURVE (8 PETALS) 2

If $\tanh x = \frac{2}{5}$, find $\operatorname{sech} x$ using identities. **Do NOT use the formula for $\tanh^{-1} x$.** **SCORE: ____ / 15 PTS**

$\cosh^2 x - \sinh^2 x = 1$

6 $1 - \tanh^2 x = \operatorname{sech}^2 x$

$1\frac{1}{2}$ $1 - \frac{4}{25} = \operatorname{sech}^2 x$

$\operatorname{sech}^2 x = \frac{21}{25}$ $1\frac{1}{2}$

$\operatorname{sech} x = \pm \frac{\sqrt{21}}{5}$ 2

$\operatorname{sech} x = \frac{\sqrt{21}}{5}$ SINCE $\operatorname{sech} x = \frac{1}{\cosh x} > 0$ FOR ALL x

2 2